

# A slight excess of large-scale power from moments of the peculiar velocity field

E. Macaulay,<sup>1\*</sup> H. Feldman,<sup>2</sup> P. G. Ferreira,<sup>1</sup> M. J. Hudson<sup>3,4,5</sup> and R. Watkins<sup>6</sup>

<sup>1</sup>Astrophysics, University of Oxford, Denys Wilkinson Building, Keble Road, Oxford OX1 3RH

<sup>2</sup>Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, USA

<sup>3</sup>Department of Physics and Astronomy, University of Waterloo, Waterloo, ON N2L3G1, Canada

<sup>4</sup>Institut d'Astrophysique de Paris - UMR 7095, CNRS/Université Pierre et Marie Curie, 98bis boulevard Arago, 75014 Paris, France

<sup>5</sup>Perimeter Institute for Theoretical Physics, 31 Caroline St. N., Waterloo, ON N2L2Y5, Canada

<sup>6</sup>Department of Physics, Willamette University, Salem, OR 97301, USA

Accepted 2011 January 25. Received 2011 January 16; in original form 2010 October 11

### ABSTRACT

The peculiar motions of galaxies can be used to infer the distribution of matter in the Universe. It has recently been shown that measurements of the peculiar velocity field indicate an anomalously high bulk flow of galaxies in our local volume. In this paper, we find the implications of the high bulk flow for the power spectrum of density fluctuations. We find that analysing only the dipole moment of the velocity field yields an average power spectrum amplitude which is indeed much higher than the  $\Lambda$  cold dark matter ( $\Lambda$ CDM) value. However, by also including shear and octupole moments of the velocity field, and marginalizing over possible values for the growth rate, an average power spectrum amplitude which is consistent with the  $\Lambda$ CDM is recovered. We attempt to infer the shape of the matter power spectrum from moments of the velocity field, and find a slight excess of power on scales  $\sim 1 h^{-1}$  Gpc.

**Key words:** galaxies: kinematics and dynamics – galaxies: statistics – cosmology: observations – cosmology: theory – large-scale structure of Universe.

## **1 INTRODUCTION**

Peculiar velocities are useful cosmological probes. In principle, the peculiar velocity field is an unbiased tracer of the underlying matter distribution and should be sensitive to structures on scales larger than the nominal size of the survey. The study of large-scale flows has a long and hallowed tradition; throughout the 1980s and early 1990s, measurements of large-scale flows were deployed to not only constrain the fractional energy density in matter,  $\Omega_m$ , but also identify possible sources of the gravitational attraction which might lie outside current galaxy surveys (Strauss & Willick 1995).

Velocity catalogues are hard to construct. While current redshift surveys have of the order of  $10^5$  objects, velocity catalogues are typically restricted to of the order of  $10^3$  galaxies. Yet over the past three decades, enough measurements of peculiar velocities have been accrued to be able to construct a reasonably complete catalogue out to a maximum distance of about  $100 h^{-1}$  Mpc. In a recent series of papers (Watkins, Feldman & Hudson 2009; Feldman, Watkins & Hudson 2010), the authors showed that a collection of peculiar velocity surveys could be combined to construct a reliable, wellbehaved 'composite' catalogue. This composite catalogue was then

\*E-mail: edward.macaulay@astro.ox.ac.uk

used to extract the simplest statistic: the bulk flow. The results were surprising: there is clear evidence for a large bulk flow, which is very unlikely within the current preferred model of the structure formation, that is, a flat, Friedman-Robertson-Walker universe with an appreciable cosmological constant permeated by Gaussian, scaleinvariant perturbations [known as the  $\Lambda$  cold dark matter ( $\Lambda$ CDM) model]. The findings of Watkins et al. (2009) and Feldman et al. (2010) lead to the question: is there excess power on large scales? As yet we do not have a direct measurement of the power spectrum of density fluctuations on these scales in the nearby Universe; the cosmic microwave background does probe a wide range of scales but at high redshift. Could the large bulk flow be a signature for large-scale fluctuations at low redshift? It is this question we wish to address by using the measurement of the bulk flow and a few of the other lower moments to estimate the power spectrum of density fluctuations.

Jaffe & Kaiser (1995) inferred constraints on the matter power spectrum parametrized by  $\sigma_8$  and the shape parameter  $\Gamma$  with data from Lauer & Postman (1994), and separately with data from Riess, Press & Kirshner (1995). They found that including only the bulk flow of the Lauer & Postman catalogue favoured a model with high large-scale power, which became more reasonable when the shear of the velocity field was included, suggesting a very large scale excess of power. However, the data from Riess et al. (1995) did not suggest any large-scale excess. The shear and dipole moments used by Jaffe & Kaiser (1995) are slightly different from the minimum variance shear and dipole moments we use here, because the Jaffe & Kaiser (1995) method assumes that the non-modelled velocity is noise. Thus, the dipole moment from the Jaffe & Kaiser (1995) method will depend on whether the shear is included, whereas the minimum variance formalism we employ here estimates each moment independently of the total number of moments analysed. Further, Jaffe & Kaiser (1995) compare very different catalogues with different geometries, that is, densities, distribution, etc. The minimum variance method combines individual measurements to estimate the velocity moments for an idealized geometry and the results are thus independent of the geometry of any particular survey.

Kolatt & Dekel (1995) inferred constraints on the large-scale structure from the POTENT velocity field. Uncertainty in the growth of the structure rate, f, was included by reporting the degenerate combination of  $P(k) f^2$ . With current improved constraints on the growth rate, and more peculiar velocity data available, we attempt to constrain the unbiased matter power spectrum from moments of the peculiar velocity field.

Recently, Song et al. (2010) applied a method to estimate bulk flows from two-point correlations in galaxy distributions and found no evidence for a large bulk flow in data from the SDSS. However, their analysis only probes scales up to 0.03 h Mpc<sup>-1</sup>, whereas we see later that the anomalously high bulk flow is most sensitive to scales >0.01 h Mpc<sup>-1</sup>.

This paper is structured as follows. In Section 2, we review the methodology to incorporate an individual peculiar velocity measurement into an estimate of our local peculiar velocity field. We review the method of Kaiser (1988) to weigh peculiar velocity measurements to estimate the bulk flow. We also review the method of Watkins et al. (2009) to use peculiar velocity measurements to estimate the bulk flow of our local volume, so that results from different surveys can be directly compared. We describe the peculiar velocity catalogues compiled by Watkins et al. (2009). We then review the work of Feldman et al. (2010) to include higher moments of the velocity field. In Section 3, we describe how we relate these moments of the velocity field to constraints on the large-scale structure. We present our results in Section 4.

# 2 COMBINING PECULIAR VELOCITY MEASUREMENTS

To estimate the peculiar velocity of galaxies at cosmological distances, we need both the measured redshift of the galaxy and an independent distance measure, such as the luminosity distance. From the distance measure, we can estimate what we would expect the redshift to be solely due to the Hubble flow. We can then attribute the difference between this expected redshift and the measured redshift to the peculiar velocity of the galaxy (Peebles 1993). There are two key difficulties with the method. The first is that the uncertainty on the luminosity distance is typically rather large:  $\sim 10$  to 20 per cent for Tully-Fisher, Faber-Jackson or Fundamental Plane measurements, and  $\sim$ 5 per cent for supernovae. The second difficulty is more inherent: this method only provides the line-of-sight component of the peculiar velocity. The importance of this effect can be illustrated if we consider a hypothetical bulk flow from an arbitrary north to the south. A survey of galaxies to our north or south would be sensitive to this flow, whereas a survey to our east or west would not. The approach we describe here is to combine individual peculiar velocity measurements to estimate the velocity field of our local volume by weighing each measurement according to how sensitive it is to each component of the underlying velocity field. We can describe the velocity field v(r), where v is the three components of the peculiar velocity field at position r. For each galaxy (labelled n), we measure the line-of-sight component of this field, S, that is,  $S_n(r) = v(r_n) \cdot \hat{r}_n$ , where  $\hat{r}_n$  is the unit vector pointing in the  $r_n$  direction. We assume that this measurement is drawn from a Gaussian distribution with variance  $\sigma_n^2$ . An additional term,  $\sigma_*^2$ , is included in the variance to account for non-linear flows. The method is fairly insensitive to the particular choice of  $\sigma_*^2$ , as the combined uncertainty of  $\sigma_n^2 + \sigma_*^2$  tends to be dominated by the measurement uncertainty,  $\sigma_n^2$ .

The simplest result we can quote for a survey of peculiar velocities is the 'bulk flow' vector,  $\boldsymbol{u}$ , the average velocity magnitude and the direction of the galaxies in a survey. We must be careful when combining peculiar velocity measurements to include the effect of only measuring the line-of-sight component. This is achieved by multiplying each component of  $S_n$  (in the coordinate system of the bulk flow) by a weight  $w_a$ , which essentially depends on the orthogonality of S to the coordinate system of the bulk flow, so that  $u_a = \sum_n w_{a,n} S_n$ . Kaiser (1988) showed that the weights for the bulk flow are

$$w_{i,n} = A_{ij}^{-1} \sum_{m} \frac{\hat{\boldsymbol{x}}_{j} \cdot \hat{\boldsymbol{r}}_{n}}{\sigma_{n}^{2} + \sigma_{*}^{2}},\tag{1}$$

where

$$A_{ij} = \sum_{m} \frac{(\hat{\boldsymbol{x}}_i \cdot \hat{\boldsymbol{r}}_m)(\hat{\boldsymbol{x}}_j \cdot \hat{\boldsymbol{r}}_m)}{\sigma_m^2 + \sigma_*^2}.$$
(2)

Because of sparse sampling, bulk flow results between different surveys are not necessarily immediately comparable. Watkins et al. (2009) devised a method to weigh the bulk flow of a particular survey to estimate the bulk flow of a hypothetical spherically symmetric survey, where the density of galaxies *n* falls off as  $n(r) \propto \exp(-r^2/2R_I^2)$ , where  $R_I^2$  is the characteristic depth of the survey. This allows bulk flows from different surveys to be directly compared and combined to give us a better estimate of the bulk flow of our local volume of space. We start with the formalism from Kaiser (1988) and then introduce the method from Watkins et al. (2009).

The idea is that the bulk flow we measure,  $\boldsymbol{u}$ , is essentially a convolution of the peculiar velocity field  $\boldsymbol{v}$  with the window function of the survey W:

$$u_i(\boldsymbol{r}_0) = \int \mathrm{d}^3 \boldsymbol{r} \, W_{ij}(\boldsymbol{r}) v_j(\boldsymbol{r} + \boldsymbol{r}_0). \tag{3}$$

This allows us to calculate the variance of u directly from the convolution theorem:

$$\left\langle u_{i}u_{j}\right\rangle =\int\mathrm{d}^{3}k\,P_{v}(k)W_{ij}^{2}(k),\tag{4}$$

where the window function is given (in real space) by

$$W_{ij}(r) = A_{ij}^{-1} \sum_{n} \delta(r - r_n) \frac{\hat{r}_i \hat{r}_j}{\sigma_n^2 + \sigma_*^2}.$$
 (5)

It is extremely useful to split  $\langle u_i u_j \rangle$  into two terms: one term,  $R_{ij}^{(v)}$ , due to the measurement variance, and a noise term,  $R_{ij}^{(\epsilon)}$  due to the variance from non-linear flows, so that  $\langle u_i u_j \rangle = R_{ij}^{(v)} + R_{ij}^{(\epsilon)}$ . We then have

$$R_{ij}^{(\epsilon)} = A_{ij}^{-1} \tag{6}$$

and

$$R_{ij}^{(v)} = \int d^3k \mathcal{W}_{ij}^2(k) P_v(k),$$
(7)

 $\hfill \ensuremath{\mathbb{C}}$  2011 The Authors, MNRAS **414**, 621–626 Monthly Notices of the Royal Astronomical Society  $\hfill \ensuremath{\mathbb{C}}$  2011 RAS

where

$$W_{ij}^2 = W_{il}(k)W_{jm}^*(k)\hat{k}_l\hat{k}_m.$$
 (8)

We will return to equations (6) and (7) when we come to estimate the large-scale structure in Section 3.

The approach of Watkins et al. (2009) is to find weights that minimize the average variance between a particular bulk flow vector  $\boldsymbol{u}$  and the average bulk flow of our hypothetical survey,  $\boldsymbol{U}$ ,  $\langle (u_i - U_i)^2 \rangle$ . If we assume that the measurement error is uncorrelated with the bulk flow, we can expand  $\langle (u_i - U_i)^2 \rangle$  as

$$\left\langle (u_i - U_i)^2 \right\rangle = \sum_{n,m} w_{i,n} w_{i,m} \left\langle S_n S_m \right\rangle + \left\langle U_i^2 \right\rangle - 2 \sum_n w_{i,n} \left\langle U_i v_n \right\rangle.$$
(9)

Before minimizing this expression with respect to  $w_{i,n}$ , we enforce the constraint that  $W_{ij}^2(k) \rightarrow 1/3$  as  $k \rightarrow 0$ , so that the weighing is equal for each dimension of the peculiar velocity field. We thus have to minimize

$$\sum_{n,m} w_{i,n} w_{i,m} \langle S_n S_m \rangle + \langle U_i^2 \rangle -2 \sum_n w_{i,n} \langle u_i U_i \rangle \lambda \left( P_{nm} w_{i,n} w_{i,m} - \frac{1}{3} \right), \qquad (10)$$

where  $\lambda$  is a Lagrange multiplier and

$$P_{nm} = \int \frac{\mathrm{d}^2 \hat{k}}{4\pi} \left( \hat{r}_n \cdot \hat{k} \hat{r}_m \cdot \hat{k} \right). \tag{11}$$

Differentiating equation (10) and setting the result equal to zero gives

$$\sum_{m} (\langle S_n S_m \rangle + \lambda P_{nm}) w_{i,m} = \langle S_n U_i \rangle.$$
(12)

It is easier to solve for the weights,  $w_{i,m}$ , if we rewrite equation (12) in the matrix form. Substituting **G** for  $\langle S_n S_m \rangle$ , **P** for  $P_{nm}$ , **w**<sub>i</sub> for  $w_{i,m}$  and **Q**<sub>i</sub> for  $\langle S_n U_i \rangle$  gives

$$(\mathbf{G} + \lambda \mathbf{P})\mathbf{w}_i = \mathbf{Q}_i,\tag{13}$$

which is now easy to solve for the weights  $\mathbf{w}_i$ .

$$\mathbf{w}_i = (\mathbf{G} + \lambda \mathbf{P})^{-1} \mathbf{Q}_i. \tag{14}$$

As well as the bulk flow of the survey, we can also consider higher moments of the peculiar velocity field if we consider the field as a Taylor expansion, as in

$$v_i(\mathbf{r}) = U_i + U_{ij}r_j + U_{ijk}r_jr_k + \cdots, \qquad (15)$$

where  $U_i$  is the bulk flow vector – also known as the 'dipole moment' and  $U_{ij}$  is the 'shear tensor' or the 'quadrupole moment' and provides information about the distance at which the bulk flow attractor is located. The 'octupole tensor'  $U_{ijk}$  provides information about the velocity field on scales smaller than the survey. Feldman et al. (2010) have extended the work of Watkins et al. (2009) to include these higher order moments. If we assume that the peculiar velocity field is entirely due to the gravitational infall, we expect the field to be curl-free and consequently  $U_{ij}$  and  $U_{ijk}$  to be symmetric. Thus, the peculiar velocity field can be described to third order by the 19 independent velocity moments,  $U_i$ .

Before we proceed, we have to be careful with the definition of the octupole tensor, because some components of the tensor overlap with the dipole moment. As such, we modify the expansion of the velocity field to be

$$v_i(\mathbf{r}) = U_i + U_{ij}r_j + U_{ijk}\left(r_jr_k - \Lambda_{jk}\right) + \cdots, \qquad (16)$$

© 2011 The Authors, MNRAS **414**, 621–626 Monthly Notices of the Royal Astronomical Society © 2011 RAS where  $\Lambda_{jk}$  is given by

$$\Lambda_{jk} = \int_{V} r_j r_k \mathrm{d}^3 r \tag{17}$$

in order to remove overlapping components. The line-of-sight component is then

$$s(\mathbf{r}) = U_i \hat{r}_i + U_{ij} r \hat{r}_i \hat{r}_j + U_{ijk} \left( r^2 \hat{r}_i \hat{r}_j \hat{r}_k - \Lambda_{jk} \hat{r}_i \right) + \cdots .$$
(18)

Feldman et al. (2010) have calculated the minimum variance weights for the 19 components of the third-order expansion, over k ranges from 0.002 to 0.196 h Mpc<sup>-1</sup>. We can think of the resulting set of 19 components of the dipole, quadropole and octupole as a form of data compression containing the highest signal-to-noise ratio information in a given peculiar velocity survey. Indeed, that will be our philosophy – to use this form of the data to infer information about the underlying density field.

We study the 'COMPOSITE' peculiar velocity catalogue compiled by Watkins et al. (2009) and also used in Feldman et al. (2010). The catalogue consists of 4536 peculiar velocity measurements, with a characteristic depth of  $34 h^{-1}$  Mpc. The characteristic depth is given by the average distance to each galaxy, weighed by the inverse square of the peculiar velocity uncertainty.

#### **3 RELATING VELOCITY TO MATTER**

We have presented a method to combine line-of-sight estimates of the peculiar velocity of individual galaxies into an estimate of the moments describing the velocity field of our local volume. We now want to be able to compare this measurement to expectations from our cosmology. The basic idea is that, in the linear regime, galaxies flow towards local overdensities of matter, so the velocity field v(r)is given by

$$\boldsymbol{v}(\boldsymbol{r}) = \frac{fH_0}{4\pi} \int d^3 \boldsymbol{r}' \delta(\boldsymbol{r}') \frac{(\boldsymbol{r}' - \boldsymbol{r})}{|\boldsymbol{r}' - \boldsymbol{r}|^3},\tag{19}$$

where f is the perturbation growth rate,  $\partial \ln \delta / \partial \ln a$ , and  $\delta$  is the matter density contrast. The matter density contrast is modelled as a Gaussian random field with a power spectrum defined as  $\langle |\tilde{\delta}(k)|^2 \rangle = (2\pi)^3 P(k)$ , where  $\tilde{\delta}$  is the Fourier transform of the density contrast in real space.

A useful way of parametrizing P(k) is in terms of band powers:

$$P(k) = \begin{cases} P_{\alpha} & k_{\alpha} < k < k_{\alpha+1} \\ 0 & \text{otherwise.} \end{cases}$$
(20)

To estimate the most likely matter power spectrum, based on the peculiar velocity data, our approach is to construct, and then minimize, the likelihood function  $\mathcal{L}$  as a function of  $P_{\alpha}$ :

$$-2\ln \mathcal{L}(P_{\alpha}) \propto \ln |\mathbf{C}| + \boldsymbol{u}^{\mathrm{T}} \mathbf{C}^{-1} \boldsymbol{u}, \qquad (21)$$

where  $\boldsymbol{u}$  are the velocity moment components. The covariance matrix,  $\boldsymbol{C}$ , is derived from equation (4):

$$\mathcal{C}_{pq} = R_{pq}^{(v)} + R_{pq}^{(\epsilon)},\tag{22}$$

where the 'error matrix'  $R_{pq}^{(\epsilon)}$  is given by equation (6) and the 'velocity matrix'  $R_{pq}^{(v)}$  is given by equation (7). We relate the velocity covariance matrix to the power spectrum by equation (7), so that

$$R_{pq}^{(v)} = \frac{f^2}{2\pi^2} \int dk P(k) \mathcal{W}_{pq}^2(k).$$
(23)

We can choose the width of each band power by integrating the window function (which is independent of  $P_{\alpha}$ ) over the bin range



**Figure 1.** Likelihood distributions for the average P(k) value in the *k* range of our window function, inferred from a *simulated* peculiar velocity catalogue. Whether we include only the bulk flow of the velocity field (labelled 'Dipole') or also include the shear and octupole moments (labelled 'Octupole'), we recover a value which is compatible with the fiducial  $\Lambda$ CDM value. For clarity, the likelihood functions have been scaled arbitrarily to the maximum value of the dipole function. The  $\Lambda$ CDM value has been illustrated here and in further such plots as a Gaussian distribution with an arbitrary standard deviation of 10 per cent of the fiducial value.

 $k_{\alpha}-k_{\alpha+1}$ . We can then factor in the average power amplitude in this range, so that

$$R_{pq}^{(v)} \simeq \frac{f^2}{2\pi^2} \sum_{\alpha} P_{\alpha} \mathcal{K}_{pq}^{\alpha}, \tag{24}$$

where the kernel  $\mathcal{K}$  is given by

$$\mathcal{K}^{\alpha}_{pq} = \int_{k_{\alpha}}^{k_{\alpha}+1} \mathrm{d}k \mathcal{W}^{2}_{pq}(k).$$
<sup>(25)</sup>

We thus have a likelihood function for the velocity moments in terms of the growth rate and power spectrum band-powers.

We have tested our methodology on simulated peculiar velocity catalogues generated from a  $\Lambda$ CDM power spectrum. We generate a series of mock catalogues from a set of *N*-body simulations, estimate the corresponding dipole, quadropoles and octopoles, and then minimize the corresponding likelihood to recover the input power spectrum. We illustrate our results for one such realization in Figs 1 and 2, and confirm that our method does not generate any spurious large-scale power.

#### **4 RESULTS AND DISCUSSION**

We first consider the most likely total average power of our sample, over the entire *k* range of our window function, 0.002-0.196 h Mpc<sup>-1</sup>. We see in Fig. 3(a) that when we include only the dipole moment of the velocity field, the average power for each survey is much higher than the  $\Lambda$ CDM value. This is entirely consistent with the results found in Watkins et al. (2009). When we also include the shear and octupole moments, we find much better agreement with the  $\Lambda$ CDM value, as we can see in Fig. 3(b). This is similar to the effect observed by Jaffe & Kaiser (1995) when including the shear of the peculiar velocity field and also noted by Feldman et al. (2010).

In Fig. 3, the growth rate was fixed at the fiducial  $\Lambda$ CDM value to illustrate the importance of including higher moments of the velocity field. However, presently, the best constraints on the growth



**Figure 2.** In the upper figure is plotted the power spectrum shape, inferred only from peculiar velocity data generated from a simulated catalogue. We have chosen three bands, evenly spaced in log *k*. The shaded region represents the marginalized  $1\sigma$  uncertainty on the power in each band. For comparison, the theoretical  $\Lambda$ CDM power spectrum has been overplotted. The results shown here include the dipole, shear and octupole moments of the simulated velocity field. As discussed later, the uncertainties in each band have also been marginalized over the growth rate. In the lower panel are plotted the average values of the dipole, shear and octupole window functions. The dipole is most sensitive on scales  $k \leq 0.01 h \, \text{Mpc}^{-1}$ .

rate at low redshift are from Peacock et al.'s (2001) measurement of the redshift-space distortion compression parameter. This constrains the growth rate to  $f = 0.49 \pm 0.14$ . We now consider the effect of marginalizing over the growth rate with this prior applied. Likelihood contours for the growth rate and average power in the COMPOSITE survey are shown in Fig. 4. When we marginalize over the growth rate, we see in Fig. 5 that the average power does not exclude the fiducial  $\Lambda$ CDM value. However, in the context of  $\Lambda$ CDM models, the growth rate is not a free parameter, rather it is determined primarily by  $\Omega_m$  (which also affects the shape of the power spectrum and the average power). In order to obtain consistency with the predicted average power, one requires  $f \sim 0.7$ , which is much larger than the fiducial  $\Lambda$ CDM value of 0.48.

In addition to the total average value, we can attempt to infer the shape of the power spectrum. The window function was divided into three ranges, evenly spaced in  $\log k$ . The likelihood function was then evaluated in terms of the power in each of the three bins,  $P_{\alpha}$ , and the growth rate. As before, a prior on the growth rate was applied and this parameter was marginalized. When we consider only the bulk flow, we find that the likelihood is sufficiently broad and correlated that it is, in practice, impossible to pin down three independent estimates of  $P_{\alpha}$ . This is disappointing – the measurement of the dipole alone does not allow us to identify if there is large-scale power.

However, when we also include the shear and octupole moments of the velocity field, we find that the band powers are virtually uncorrelated and we can obtain three independent measurements. In Fig. 6, we plot the marginalized  $1\sigma$  uncertainty for each  $P_{\alpha}$  for the COMPOSITE catalogue. There is a slight detection of excess power on the largest scales.

We find that our estimate of the excess power seems to reflect the large-scale estimates of P(k) found in the estimate from both the SDSS main galaxy power spectrum from Percival et al. (2007)



**Figure 3.** In the upper panel is the likelihood distribution for the average amplitude of the power spectrum, inferred only from the dipole moment. To account for the anomalously high bulk flow, we have to conclude a power spectrum amplitude which is incompatibly higher than the  $\Lambda$ CDM value. In the lower panel, we show the effect of including the shear and octupole moments of the velocity field. By now including the higher moments, the results are generally more compatible with the  $\Lambda$ CDM value. Here, the growth rate, *f*, is kept fixed at the fiducial  $\Lambda$ CDM value. In both figures, the shaded area is the 1 $\sigma$  confidence region. The shaded area of the upper figure appears disproportionately wide, because the large tail of the distribution is distorted by the logarithmic scale on the horizontal axis.

and the 2dFGRS power spectrum from Cole et al. (2005). One should be careful about overinterpreting these similarities, as both the SDSS and 2dFGRS estimates of the power on large scales are, in principle, severely affected by edge effects. Indeed, the recent analysis of the SDSS data in Sylos Labini et al. (2009) should make one to be wary of overinterpreting estimates of the structure on the largest scales of the survey. Yet the fact that the same level of fluctuations is obtained from a peculiar velocity survey subjected to a completely different analysis may be an indication that redshift survey estimates of the structure on the largest scale must be taken seriously. Furthermore, an excess of clustering on similar scales at higher redshift was recently found by Thomas, Abdalla & Lahav (2010) in the MegaZ DR7 photometric redshift survey.



**Figure 4.** Likelihood contours for the COMPOSITE survey, with dipole, shear and octupole moments included, with contours spaced at 1, 2 and  $3\sigma$  confidence levels. We are now considering the average power and the perturbation growth rate as free parameters. A prior on the growth rate has been applied. For comparison, the one-dimensional likelihood distribution for the COMPOSITE survey in Fig. 3(b) is a slice through this likelihood contour at the fiducial  $\Lambda$ CDM value of the growth rate at z = 0, that is, f = 0.48. As can be seen here, including observationally constrained values of the growth rate allows for a wider range of allowed  $P_{\alpha}$ .



**Figure 5.** Average power for the COMPOSITE survey, with dipole, shear and octupole moments included, now marginalized over the growth rate. The shaded region indicates the  $1\sigma$  uncertainty boundaries on the likelihood function. The expected  $\Lambda$ CDM value is now included in this range.

What should we make of this excess of power and how does it relate to our initial question: the anomalously high bulk flow? To start with, we have demonstrated that it is incomplete to infer results from measurements of the bulk flow alone, as including higher moments of the velocity field yields significantly different results. As noted in Feldman et al. (2010), the low shear moments of the velocity field suggest that the sources responsible for the bulk flow are very far away. In the formalism we have adopted here, only one of the bulk flow moments is anomalously high – the other bulk flow moments, and the shear and octupole moments, are consistent with the  $\Lambda$ CDM. From one perspective, one could argue that the extra freedom from including 19 moments of the velocity field, and the growth rate, as opposed to just three moments of the dipole moment, provides a way to interpret the anomalously high bulk



**Figure 6.** Power spectrum shape inferred from dipole, shear and octupole moments for the COMPOSITE catalogue. The shaded regions are  $1\sigma$  uncertainties, marginalized over the growth rate, and the other  $P_{\alpha}$ . We find a slight excess of power on scales of  $\sim 1 h^{-1}$  Gpc. These are the scales at which we are most sensitive. This excess of power appears to agree with the largest scales in the 2dFGRS and SDSS, which are plotted here as in Percival et al. (2007) and have been deconvolved from their survey window functions.

flow which is consistent with the  $\Lambda$ CDM. On the other hand, all moments are not equivalent. The dipole moments are special in the sense that they probe the largest scales (and are also the moments that are most robustly measured). Consequently, the inferred shape of the power spectrum is different from the  $\Lambda$ CDM.

# ACKNOWLEDGMENTS

We would like to thank Joe Zuntz and Andrew Jaffe for useful discussions. PGF acknowledges support from STFC, BIPAC and the Oxford Martin School. HF has been supported in part by an NSF grant AST-0807326 and by the University of Kansas General Research Fund. MJH has been supported by NSERC and acknowledges the hospitality of the Institut d'Astrophysique de Paris and the financial support of the IAP/UPMC visiting programme and the French ANR (OTARIE).

#### REFERENCES

- Cole S. et al., 2005, MNRAS, 362, 505
- Feldman H. A., Watkins R., Hudson M. J., 2010, MNRAS, 407, 2328
- Jaffe A. H., Kaiser N., 1995, ApJ, 455, 26
- Kaiser N., 1988, MNRAS, 231, 149
- Kolatt T., Dekel A., 1995, preprint (arXiv:astro-ph/9512132)
- Lauer T., Postman M., 1994, ApJ, 425, 418
- Peacock J. A. et al., 2001, Nat, 410, 169
- Peebles P. J. E., 1993, Principles of Physical Cosmology. Princeton Univ. Press, Princeton, NJ
- Percival W. J. et al., 2007, ApJ, 657, 645
- Riess A. G., Press W. H., Kirshner R. P., 1995, ApJ, 445, L91
- Song Y., Sabiu C. G., Nichol R. C., Miller C. J., 2010, J. Cosmol. Astropart. Phys., 1, 25
- Strauss M., Willick J., 1995, Phys. Rep., 261, 271
- Sylos Labini F., Vasilyev N. L., Baryshev Y. V., López-Corredoira M., 2009, A&A, 505, 981
- Thomas S. A., Abdalla F. B., Lahav O., 2010, preprint (arXiv:1012.2272)
- Watkins R., Feldman H. A., Hudson M. J., 2009, MNRAS, 392, 743

This paper has been typeset from a TEX/LATEX file prepared by the author.